# Probstat - Clex 7 - Solutions



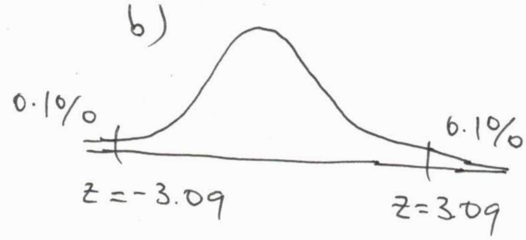
(1) a) $X = 50$, $z = \dfrac{50 - 46.8}{1.75} = 1.83$, $p(X < 50) = p(z < 1.83) = 0.9664$

b) $X = 48$, $z = \dfrac{48 - 46.8}{1.75} = 0.69$, $p(X \geq 48) = p(z \geq 0.69) = 1 - 0.7549 = 0.2$

(2) a) $p(X = 105) = 0$ {or, it is a silly question}.

$X = 105$, $Z = \dfrac{105 - 104}{5} = 0.2$, $p(X < 105) = p(X \leq 105) = p(z < 0.2) = 0.579$

b)




$X_{lower} = 104 - 3.09(5) = 88.55$

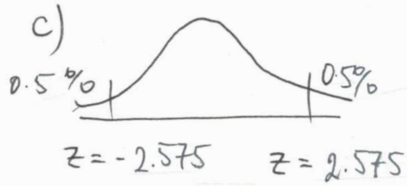
$X_{upper} = 104 + 3.09(5) = 119.45$

(3) a) $X = 14$, $Z = \dfrac{14 - 21}{4} = -1.75$; $P(X < 14) = P(z < -1.75) = 0.0401$

b) $X = 11$, $z = \dfrac{11 - 21}{4} = -2.5$; $X = 31$, $z = \dfrac{31 - 21}{4} = 2.5$

$P(11 < X < 31) = P(-2.5 < Z < 2.5) = 0.9876$

c)




$P\left(-\dfrac{10}{\sigma} < Z < \dfrac{10}{\sigma}\right) = 0.99 \Rightarrow P\left(Z < \dfrac{10}{\sigma}\right) = 0.995$

$\dfrac{10}{\sigma} = 2.575 \Rightarrow \sigma = 3.8835$ days

d) $P(X > 32) = P(Z > 2.75) = 0.0030$

Y - The number of people who take more than 32 days to recover,

$Y \sim \text{Binomial}(5, 0.003)$

$P(Y \geq 2) = 1 - [P(Y = 0) + P(Y = 1)] = 1 - [{}_5C_0 (0.003)^0 (0.997)^5 + {}_5C_1 (0.003)^1 (0.997)^4]$

$= 8.95 \times 10^{-5}$

e) $P(Z < Z_0) = 0.999 \Rightarrow Z_0 = 3.08$; $X = \mu + Z_0 \sigma = 21 + (3.08)4 = 33.32$ days

(4) a) $P(X > 40.67) = P(Z > 0.65) = 0.2578$

b) $P(X < 22.99) = P(Z < -3.22) = 0.0006$

c) $P(-Z_0 < Z < Z_0) = 0.9 \Rightarrow Z_0 = 1.28$

$X = \mu \pm Z_0 \sigma = 37.69 \pm (1.28)(4.57) \Rightarrow (31.84, 43.54)$

d) $P(X < 28.54) = P(Z < -2) = 0.0228$

Y - number of suits that cost less than 28.54: $Y \sim Binomial(10, 0.0228)$

$P(Y \leq 2) = P(Y=0) + P(Y=1) + P(Y=2) =$

$= {}_{10}C_0 (0.0228)^0 (0.9772)^{10} + {}_{10}C_1 (0.0228)^1 (0.9772)^9 + {}_{10}C_2 (0.0228)^2 (0.9772)^8$

$= 0.9988$

e) $P(Z < Z_0) = 0.95 \Rightarrow Z = 1.645$, $\mu = X - Z\sigma = 30 - 1.645(4.57) = 22.48$

5 Hypergeometric: $p = \frac{5800}{36000} = 0.1611$, $\mu = np = (1100)(0.1611) = 177.22$

$\sigma^2 = np(1-p)\frac{N-n}{N-1} = 144.131$, $\sigma = 12.0055$

$X = 175$, $Z = \frac{174.5 - 177.22}{12.0055} = -0.23$ ; $X = 180$, $Z = \frac{180.5 - 177.22}{12.0055} = 0.27$

$P(175 \leq X \leq 180) \approx P(-0.23 < Z < 0.27) = 0.6064 - 0.4090 = 0.1974$

6 Poisson: $\lambda = 6 \cdot 8 = 48$; $X = 45$, $Z = \frac{44.5 - 48}{\sqrt{48}} = -0.51$; $X = 50$, $Z = \frac{50.5 - 48}{\sqrt{48}} = 0.36$

$P(45 \leq X \leq 50) = P(-0.51 \leq Z \leq 0.36) = 0.6406 - 0.3050 = 0.3356$

$P_{Poisson} = 0.335556$