PROBABILITY AND STATISTICS, CLASS EXERCISE 8

(1) Suppose that X is a random variable with pdf

$$p(x) = \frac{1}{(1+x)^2}, \quad x \geq 0$$

Determine pdf and the cdf of $Y = \sqrt{X}$.

(2) Buy orders come to crypto currency exchange according to a Poisson process with rate of 4.7 orders per second.

a) Determine the pdf for the waiting time for fortieth buy order.

b) Determine the mean and the variance for the waiting time for the fortieth buy order.

c) Determine the probability that more than 10 seconds will ellapse before the fortieth order is placed on the exchange. (Use software)

For problems 3-5 use software and simply write down the answers.

(3) Let X be a t-distrubuted RV with 24 degrees of freedom; $X \sim Stud(24)$. Compute

a) $p(X > 1.22)$

b) $p(-0.78 < X < 2.04)$

c) The X-value which corresponds to the 95th percentile of the distribution.

d) Sketch the pdf of X.

(4) Let X be a Gamma-distrubuted RV with scale $\lambda = 3.24$ and shape $r = 7.41$; $X \sim Gamma(3.24, 7.41)$. Compute

a) $p(X < 4.23)$

b) $p(2.8 < X < 3.4)$

c) The X-value which corresponds to the 95th percentile of the distribution.

d) Sketch the pdf of X

(5) Let X be a χ^2-distrubuted RV with k=11 degrees of freedom; $X \sim \chi^2(11)$. Compute

a) $p(X > 7.23)$

b) $p(8.0 < X < 12.4)$

c) The X-value which corresponds to the 95th percentile of the distribution.
d) Sketch the pdf of X.

(6) Let $X \sim (0, \sigma)$ be centered, but nonstandard normal. A special case of the *folded normal* distribution is the distribution of $Y = |X|$. Determine the pdf of the Y and its expected value. Sketch the pdf of Y.

(7) Random variable X has the pdf $p(x) = 4x(1 - x^2),\ 0 \leq x \leq 1$. Determine $E(X^2)$ in two ways:
a) Without finding the pdf of Y.
b) By first computing the pdf of Y.