

# Probstat - Clex 9 - solutions

⑩

①  $X \mid \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline P(X) & 1/4 & 1/2 & 1/4 \end{array}$       $\mu_X = 1 \cdot 1/2 + 2 \cdot 1/4 = 1$  ;  $\sigma_X^2 = 1^2 \cdot 1/2 + 2^2 \cdot 1/4 - 1 = 1/2$   
 $\sigma_X = \frac{1}{\sqrt{2}}$  ;  $\mu_{\bar{X}} = 1$  ;  $\sigma_{\bar{X}} = \frac{1}{\sqrt{100 \cdot 2}} = \frac{1}{10\sqrt{2}}$

$\bar{X} = 0.9$ ,  $Z = \frac{0.9 - 1}{1/10\sqrt{2}} = -1.41$  ;  $\bar{X} = 1.1$ ,  $Z = 1.41$

$P(0.9 < \bar{X} < 1.1) = P(-1.41 < Z < 1.41) = 0.84146$

②  $\bar{X} = \frac{1}{5} (468 + 449 + 494 + 496 + 479) = 477.2$

$S^2 = \frac{1}{4} [(468 - 477.2)^2 + \dots + (479 - 477.2)^2] = 379.7$

$\frac{(n-1)S^2}{\chi^2_{0.025}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{0.975}}$       $\frac{4 \cdot (379.7)}{11.1433} \leq \sigma^2 \leq \frac{4 \cdot (379.7)}{0.4844}$

$136.2971 \leq \sigma^2 \leq 3135.4252$

$11.67 \leq \sigma \leq 55.99$  min with 95% confidence.

③  $\mu_X = 3.5$ ,  $\sigma_X^2 = \frac{1}{6} [(1-3.5)^2 + \dots + (6-3.5)^2] = 17.5$

$\mu_{\bar{X}} = 3.5$  ;  $\sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{12} = \frac{35/12} = 0.2431$  ;  $\sigma_{\bar{X}} = 0.493$

$\bar{X} = \frac{38}{12} = 3.1667$

$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{3.1667 - 3.5}{0.493} = -0.6761$

$P(\bar{X} < 3.1667) = P(Z < -0.6761) = 0.2495$

④  $\mu_{\bar{X}} = 150$ ,  $\sigma_{\bar{X}} = 10/\sqrt{16} = \sqrt{10}$ ,  $\bar{X} = \frac{1575}{10} = 157.5$

$Z = \frac{157.5 - 150}{\sqrt{10}} = 2.37$       $P(Z > 2.37) = 0.0089$

⑤  $\bar{X} = 86.3$ ,  $Z = \frac{86.3 - 85}{15/\sqrt{277}} = 1.44$ ,  $P(\bar{X} > 86.3) = P(Z > 1.44) = 0.0749$

⑥ a)  $Z = \frac{2}{4} = 0.5$ ,  $P(-0.5 < Z < 0.5) = 0.3829$

b)  $Z = \frac{2}{4/\sqrt{6}} = 1$ ,  $P(-1 < Z < 1) = 0.6827$

Probstat - Clex 10 - solutions

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$$\textcircled{7} \quad \chi^2_1 = \frac{(n-1)s^2}{\sigma^2} = \frac{(11) \cdot (50)}{100} = 5.5 ; \quad \chi^2_2 = \frac{11(240)}{100} = 26.4$$

$$P(5.5 < \chi^2 < 26.4) = 0.9943 - 0.0954 = 0.8989$$

$$\textcircled{8} \quad \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha}} = \frac{(21)(5.04)}{8.8972} = 11.90$$

$\sigma^2 \leq 11.90$  with 99% confidence.